Geophysics 4133
Vector and Calculus Review
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Name: ________________________________

For each of the following problems neatly show all steps of your work (partial credit may be given if your work can be easily followed). Clearly indicate your final answers and answer all parts of the question including the 'describe' and 'why' questions.

1. Apply the binomial approximation: *(8 pt.)*

   a) $(1 + 0.001)^{4.5}$

   b) $(1 + 42)^{6.7}$

   c) $(12 + 1)^{5.2}$

   d) $(1 + 70)^{2}$

2. Apply the Taylor series expansion to the second order terms: *(8 pt.)*

   a) $\sin(x)$ about $x_0 = 0$

   b) e^x (leave in general form)

3. Find the distance between the given sets of points. *(8 pt.)*

   a) $P_1 = (1, 7)$ and $P_2 = (-4, -2)$

   b) $P_1 = (6, 8, -12)$ and $P_2 = (5, 2, 3)$

   c) $P_1 = (0, 0, 0,)$ and $P_2 = (3, 2, -6)$

   d) $P_1 = (9, 2, -4)$ and $P_2 = (-3, -2, -7)$

4. Find the vector connecting the given points. Express your answer in both angle bracket notation $\langle a, b, c\rangle$ and the ijk basis $a\hat{\imath} + b\hat{\jmath} + c\hat{k}$. *(8 pt.)*

   a) $P_1 = (8, 2)$ and $P_2 = (-7, -2)$

   b) $P_1 = (9, 0)$ and $P_2 = (12, 4)$

   c) $P_1 = (2, 5, 3)$ and $P_2 = (-7, 3, -2)$

   d) $P_1 = (1, 7, -8)$ and $P_2 = (-4, 2, 3)$

5. For each of the following vectors find the magnitude and convert it to a unit vector (in bracket notation). *(16 pt.)*

a) $\vec{a} = \langle 5, 3 \rangle$

b) $\vec{a} = \langle 2, -6 \rangle$

c) $\vec{a} = \langle 1, -1, 3 \rangle$

d) $\vec{a} = \langle 0, 9, 4 \rangle$

6. Add the following vectors together and write the result in ijk format $a\hat{\imath} + b\hat{\jmath} + c\hat{k}$. *(12 pt.)*

$$\begin{cases} \vec{a_1} = \langle 3, 4 \rangle & \text{and} \quad \vec{b_1} = 1\hat{\imath} + 3\hat{\jmath} + 5\hat{k} \\ \vec{a_2} = \langle -1, 5 \rangle & \text{and} \quad \vec{b_2} = 0\hat{\imath} + -8\hat{\jmath} + 9\hat{k} \\ \vec{a_3} = \langle 0, 2 \rangle & \text{and} \quad \vec{b_3} = -1\hat{\imath} + 2\hat{\jmath} + 6\hat{k} \end{cases}$$

a) $\vec{a_1} + \vec{a_2} =$

b) $\vec{a_3} + \vec{a_2} =$

c) $\vec{a_1} + \vec{a_3} + 2\vec{a_2} =$

d) $\vec{b_1} + 5\vec{b_3} =$

e) $2\vec{b_2} + 3\vec{b_1} =$

f) $\vec{b_1} + 4\vec{b_3} =$

7. Compute the indicated dot product *(12 pt.)*

$$\begin{cases} \vec{v_1} = \langle 3, 4 \rangle \\ \vec{v_2} = \langle -1, 5 \rangle \\ \vec{v_3} = \langle 0, 2 \rangle \\ \vec{w_1} = \langle 1, -2, 7 \rangle \\ \vec{w_2} = \langle 0, -1, -3 \rangle \\ \vec{w_3} = \langle 6, 3, 4 \rangle \end{cases}$$

a) $\vec{v_1} \cdot \vec{v_2} =$

b) $\vec{v_1} \cdot \vec{v_3} =$

c) $\vec{v_3} \cdot \vec{v_2} =$

d) $\vec{w_1} \cdot \vec{w_2} =$

e) $\vec{w_1} \cdot \vec{w_3} =$

f) $\vec{w_3} \cdot \vec{w_2} =$

8. Compute the indicated cross product. *(6 pt.)*

$$\begin{cases} \vec{a} = \langle 0, -1, -3 \rangle \\ \vec{b} = \langle 6, 3, 4 \rangle \end{cases}$$

a) $\vec{a} \times \vec{b}$

b) $\vec{b} \times \vec{a}$

c) $\vec{a} \times \vec{a}$

9. Given $\vec{a} = y^3 + x^2y + 4z + 2zx$ calculate $\nabla\vec{a}$. Is the result a scalar or vector? *(5 pt.)*

10. Calculate curl $xz\hat{\imath} + yz\hat{\jmath} - y^2\hat{k}$. Is the result a scalar or vector? What is the sense of rotation of a paddle wheel placed in the field (CW or CCW)? *(6 pt.)*

11. Calculate div $x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. Is the result a scalar or vector? Is the flow convergent or divergent? Why? *(6 pt.)*

12. Describe how you would calculate the LaPlacian of $\vec{a} = y^3 + x^2y + 4z + 2zx$. $(\nabla^2\vec{a})$ *(6 pt.)*